module – I - NUMERICAL METHODS

1. Employ Taylor’s series method to find an approximate solution correct to
2. fourth decimal place for the following initial-value problem at x = 0.1 and x = 0.2 : y′ = 2y + 3ex, y(0) = 0.
3. fourth decimal place for the following initial-value problem at x = 0.1 and x =0.2 : , y(0) = 1.
4. five decimal place for the following initial-value problem at x = 0.1 & x = 0.2 : , y(0) = 1. Consider upto fourth degree terms.
5. Using the modified Euler’s method, solve
6. , y(0) = 1, in steps of 0.1 at x = 0.2. (Carry out 2 iterations at each stage).
7. y′ = log(x + y), y(1) = 2, in steps of 0.2 at x = 1.4.(Carry out 2 iterations at each stage).
8. , y(20) = 5, in steps of 0.2 at x = 20.2 and x = 20.4.
9. , y(0) = 1, in steps of 0.1 at x = 0.1. (Carry out 3 iterations).
10. Employ the Runge-Kutta method of 4th order to solve the problem
11. , y(0) = 1,in steps of 0.1 at x = 0.2.
12. , y(0) = 1, in steps of 0.2 at x = 0.4.
13. , y(0) = 1, in steps of 0.1 at x = 0.2.
14. y′ = x + y, y(0) = 1 at x = 0.2 in steps of 0.2.
15. Using Milne’s predictor-corrector method, find y when
16. x = 0.8, given , y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Apply corrector formula twice.
17. x = 4.4, given 5xy′ + y2 = 2, y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143. Apply corrector formula twice.
18. when x = 0.4 accurate upto 3 decimal places, given , y(0) = 1, y(0.1) = 1.1169, y(0.2) = 1.2773, y(0.3) = 1.5049. Apply corrector formula thrice.
19. Apply Adams-Bashforth predictor-corrector method to compute
20. y(1.4) correct to four decimal places given , y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.46649, y(1.3) = 2.7514. Apply corrector formula twice.
21. y(1.4) correct to four decimal places given , y(1) = 1, y(1.1) = 2.233, y(1.2) = 1.548, y(1.3) = 1.979. Apply corrector formula twice.
22. y(0.4) correct to four decimal places given , y(0) = 1, y(0.1) = 0.9117, y(0.2) = 0.8494, y(0.3) = 0.8061. Apply corrector formula twice.

module– II -NUMERICAL METHODS and special functions

1. Using Runge-Kutta method solve the differential equation y″ = x3 y′ + x3 y at x = 0.1, y(0) = 1, y′(0) = ½ with h = 0.1.
2. Using Runge-Kutta method of 4th order , obtain the approximate solution of y″ - x2 y′ - 2xy= 1 for x = 0.1 , x = 0, y = 1, y′ = 0 with h = 0.1.
3. Using Runge-Kutta method of 4thorder , obtain the approximate solution of y″ = xy′2 - y2 for x = 0.2 correct to four decimal places, x = 0., y = 1, y′ = 0 with h = 0.2.
4. Using Runge-Kutta method of 4th order , solve y″ = y + xy′, y(0) = 1, y′(0) = 0 to find y(0.2) and y′(0.2) with h = 0.2.
5. The angular displacement θ of a simple pendulum I given by the equation where l = 98 cm and g = 980cm/sec2. If θ = 0 and at t = 0, use Runge-Kutta method to find θ and when t = 0.2 sec.
6. Obtain the solution of the equation y″ + xy′ + y = 0 by computing the value of the dependent variable corresponding to the value 0.4 of the independent variable by applying Milnes method using the following data :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 0 | 0.1 | 0.2 | 0.3 |
| y | 1 | 0.99501 | 0.98013 | 0.95568 |
| y′ | 0 | -0.0995 | -0.19602 | -0. |

1. Obtain the solution of the equation y″ + 3xy′ - 6y = 0 by computing the value of the dependent variable corresponding to the value 0.4 of the independent variable by applying Milnes method using the following data :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 0 | 0.1 | 0.2 | 0.3 |
| y | 1 | 1.03995 | 1.13804 | 1.29865 |
| y′ | 0.1 | 0.6955 | 1.258 | 1.873 |

1. Obtain the solution of the equation 2y″ = 4x + y′ by computing the value of the dependent variable corresponding to the value 1.4 of the independent variable by applying Milnes method using the following data :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 1.1 | 1.2 | 1.3 |
| y | 2 | 2.2156 | 2.4649 | 2.7514 |
| y′ | 2 | 2.3178 | 2.6725 | 3.0657 |

1. Obtain the solution of the equation y″ + y′ = 2ex by computing the value of the dependent variable corresponding to the value 0.4 of the independent variable by applying Milnes method using the following data :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 0 | 0.1 | 0.2 | 0.3 |
| y | 2 | 2.01 | 2.04 | 2.09 |
| y′ | 0 | 0.2 | 0.4 | 0.6 |

1. Obtain the series solutionof Bessel’s differential equation leading to the Bessel function.
2. Obtain the series solution of the Legendre’s differential Equation leading to the Legendre Polynomials.
3. If α and β are distinct roots of Jn(ax) = 0, then 
4. Derive Rodrigue’s Formula for Pn(x).
5. Prove that
6. Compute P0(x), P1(x), P2(x), P3(x) and P4(x).
7. Express the following in terms of Legendre Polynomials :
8. x4 + 3x3 – x2 + 5x – 2
9. 2x3 – x2 - 3x +2
10. 3x3– x2 + 5x – 2
11. x3 + 2x2 - 4x + 5
12. x3+ 2x2 - x – 3.

module- III- COMPLEX VARIABLES – I and transformations

1. Define analytic function. Derive Cauchy – Riemann equations in cartesian form.
2. Derive C–R equations in polar form as and hence deduce that.
3. Define harmonic function. Derive Cauchy – Riemann equations in polar form.
4. If is an analytic function, then prove that u and v both satisfy the two-dimensional Laplace equation.
5. If is an analytic function, then prove that the equations and represent orthogonal families of curves.
6. Show that the following functions f(z) is analytic and hence find f′(z). (i). f(z) = z + ez (ii). f(z) = coshz (iii).f(z) = logz
7. Find an analytic function f(z) = u + iv given that
8. u = e2x (x cos 2y – y sin 2y). Also find v.
9. u = ex (x siny + y cosy). Also find v.
10. u = e-2xy sin(x2 – y2). Also find v.
11. v = (r – k2/r) sin θ, r ≠0.Also find u.

e.u =

1. Show that the given real or imaginary part is harmonic and find its harmonic conjugate. Also determine the corresponding analytic function.
   1. u = ex (x cos y – y sin y).
   2. u = (r + 1/r) cosθ, r ≠0.
   3. v = e-2y sin2x.
   4. u = e2x (x cos 2y – y sin 2y).
2. Find the analytic function f(z) = u + iv given that
3. x2 + 4xy + y2).
4. .
5. .
6. .
7. If w = φ + iψ represents the complex potential for an electric field and determine the function φ.
8. In a two-dimensional fluid flow, the stream function Find the velocity potential.
9. If the potential function is find the flux function and the complex potential function.
10. If f(z) is a holomorphic function of z, show that
11. If f(z) = u + i v is regular function and is any differential function of x, y then show that
12. If f(z) is analytic function of z, then show that .
13. State and prove Cauchy’s Theorem.
14. Derive the Cauchy’s integral formula f(a) = (1/2πi**)** [f(z)/(z-a)]dz .
15. Determine the poles of the function f(z) = z2/[(z-1)2(z+2)] and the residue at each pole.
16. By Cauchy’s Residue Theorem, evaluate  [e2z/(z+1)4]dz where C : |z| = 3.
17. Using Cauchy’s Residue Theorem, evaluate { [sin πz2+cosπz2]/(z-1)2(z-2)}dz where C : |z| = 3.
18. By Cauchy’s Residue Theorem, evaluate {[3z3+2]/(z-1)(z2+9)}dz where**(i)** C : |z-2| = 2 &**(ii)** |z| = 4.
19. Define conformal mapping. Discuss the transformation w = ez.
20. Show that the transformation w = z + (1/z),z≠ 0 transforms circles and radial lines in the z-plane into family of ellipses and family of hyperbolae respectively in the w-plane.
21. Under the transformation w = z2, find the images of
22. x – y = 1 and x2 - y2 = 1
23. straight lines parallel to x and y axes.
24. the square region bounded by the lines x = 1, x = 2, y = 1, y = 2
25. the triangular region bounded by the lines x = 1, y = 1, x + y =1
26. Define a bilinear transformation. Find the bilinear transformation and its invariant points that maps
    1. the points 1, i, -1 of the z-plane onto the points 2, i, -2 of the w-plane respectively.
    2. the points 1, i, -1 of the z-plane onto the points i, 0, -i of the w-plane respectively. Also find the image of |z | < 1 in the w-plane under this transformation..
    3. the points 0, 1, ∞ of the z-plane onto the points -5, -1, 3 of the w-plane respectively.
    4. the points i, 1, -1 of the z-plane onto the points 1, 0, ∞ of the w-plane respectively. Also find the image of |z | < 1 in the w-plane under this transformation.
    5. the points -1, i, 1 of the z-plane onto the points 1, i, -1 of the w-plane respectively.
    6. the points , i, 0 of the z-plane onto the points -1, -i, 1 of the w-plane respectively.
27. Show that under the bilinear transformation w = (z – i)/(z + i), the real axis in the z-plane is mapped into the circle |w| = 1 and that the upper half of the z-plane corresponds to the interior of this circle.

Module – IV- PROBABILITY distributions & joint probability distribution

1. Define random variable, discrete random variable, discrete probability distribution, probability mass function and cumulative distribution function of a discrete random variable.
2. Define continuous random variable, probability density funcion, cumulative distribution function of a continuous random variable.
3. If X is a random variable and k is a constant, then prove that (i)E(X+k) = E(X)+k(ii) Var(kX) = k2Var(X) (iii) Var(X+k) = Var(X).
4. A random variable X has the following distribution :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | -2 | -1 | 0 | 1 | 2 | 3 |
| P(X) | 0.1 | k | 0.2 | 2k | 0.3 | k |

Find (i) k (ii) Evaluate P(X < 1) (iii) P(-1 < X ≤ 2) (iv) Mean (v) Variance.

1. A random variable X has the following distribution :

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P(X) | 0 | k | 2k | 3k | 2k | k2 | 2k2 | 7k2 + k |

Find (i) k (ii) Evaluate P(X < 4) (iii) P(3 < X ≤ 6) (iv) P(X ≥ 5) (v) mean(vi) standard deviation.

1. A random variable X take the values -3, -2,-1,0,1,2,3 such that P(x=0) = P(x<0) and P(x= -3) =P(x= -2)=P(x= -1)=P(x= 1)=P(x= 2)=P(x= 3). Find the probability distribution.
2. Find the mean, variance and standard deviation of a binomial distribution.
3. Find the probability function for the Poisson distribution by considering it as a limiting form of the Binomial distribution. Also find the mean, variance and standard deviation of a Poisson distribution.
4. In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 100 such samples, how many would be expected to contain atleast 3 defective parts.
5. A coin is tossed 6 times. Find the probability of getting (i) exactly 1 head(ii) atmost 3 heads (iii) atleast 2 heads (iv) heads only.
6. The probability that a man aged 60 will live to 70 is 0.65. Out of 10 men, now at the age of 60, find the probability that (i) at least 7 will live up to 70 (ii) exactly 9 will live up to 70 (iii) at most 9 will live up to 70.
7. A class of 100 students contain 10 bright students. Five students from the class are picked at random. Find the probability that (i) none of the picked student is a bright student (ii) all the picked are bright students.
8. A car hire firm has 2 cars, which it hires out day by day. The demand for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the probability that on a certain day (i) neither car is used and (ii) some demand is refused.
9. Given that 2% of the fuses manufactured by a firm are defective, find by using Poisson distribution, the probability that a box containing 200 fuses has (i) at least one defective fuse (ii) 3 or more defective fuses.
10. Theprobability that an individual suffers a bad reaction from a certain injection is 0.002. Using Poisson distribution, determine the probability that out of 1000 individuals (a) exactly 3 & (b) more than 2 will suffer a bad reaction.
11. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Using Poisson distribution, find the approximate number of packets containing (i) no defective blade (ii) one defective blade and (iii) two defective blades, in a consignment of 10,000 packets.
12. The probability density functon of a continuous random variable is p(x) = kx2 , -3 ≤ x≤ 3,0 , elsewhere.Find k, mean, variance, P(x<1) and P(1<x<3).
13. The probability density functon of a continuous random variable isp(x) = kx(1-x)ex , 0 ≤ x≤1,0 , elsewhere. Find k, mean, variance and standard deviation.
14. The probability density functon of a continuous random variable isp(x) = k(1-x2), 0 ≤ x≤1, 0 , elsewhere.Find k, P(0.1≤ x≤0.2), P(x≤0.3) & P(x>0.5).
15. Find the mean and variance of the normal distribution.
16. If the life of an electric bulb is an exponential variate with mean life of 1000 hours, find the probability that a bulb will last for more than 1500 hours. If two bulbs are selected at random find the probability that (i) both (ii) atleast one will last for more than 1500 hours.
17. The life length of an electronic device is exponentially distributed with mean 3 years. What is the probability that the device will function properly after 2 years of its installation ?
18. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for (i) less than 10 minutes (ii) 10 minutes or more ?
19. The duration of a telephone conversation has been found to have an exponential distribution with mean 3 minutes. Find the probability that the conversation will last for(i) more than 1 minute (ii) less than 3 minutes.
20. The life of a T.V. tube manufactured by a company is known to have a mean of 200 months. Assuming that the life has an exponential distribution, find the probability that the life of a tube manufactured by a company is (i) less than 200 months (ii) between 100 and 300 months.
21. In a normal distribution, 31% of the items are under 45 and 8% over 64. Find the mean and standard deviation, given P(0 ≤ z < 0.5) = 0.19 and P(0 ≤ z < 1.4) = 0.42.
22. In an examination taken by 500 students, the average and standard deviation of marks obtained are 40% and 10% respectively. Assuming normal distribution, find (i) how many have scored above 60% (ii) how many will pass if 50% is fixed as the minimum for passing.(A(1) = 0.3413 and A(2) =0.4772)
23. A coffee vending machine is used to fill 80 mls of coffee to each cup. It is found that 2% of the cups filled by the machine contain less than 80 mls with a standard deviation of 0.2 ml. Assuming that the amount of coffee in the cups has a normal distribution, find the average amount of coffee in the cups. ( A(2.05) = 0.48 ).
24. Steel rods are manufactured to be 3 cms in diameter but they are acceptable if they are inside the limits 2.99cms and 3.01 cms. It is observed that 5% are rejected as oversized and 5% are rejected as undersized. Assuming that diameters are normally distributed, find the mean and the standard deviation of the distribution. (A(1.65) = 0.45).
25. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for
26. More than 2150 hours
27. Less than 1950 hours
28. More than 1920 hours and Less than 2160 hours.

(A(1.833)= 0.4664, A(1.5) =0.4332, A(2) = 0.4772).

1. The joint probability functon of two discrete random variables x and y is f(x,y) = c(2x + y), where x and y can assume all integral values such that 0 ≤ x ≤ 2, 0 ≤ y ≤ 3 and f(x,y) = 0 otherwise. Find (i) the value of c (ii) P(x = 2, y = 1) (iii) P(x ≥ 1, y ≤ 2) (iv) Marginal probability distributions of x and y.
2. A joint probability distribution is given by the following table :

|  |  |  |  |
| --- | --- | --- | --- |
| Y  X | 1 | 3 | 9 |
| 2 | 1/8 | 1/24 | 1/12 |
| 4 | 1/4 | 1/4 | 0 |
| 6 | 1/8 | 1/24 | 1/12 |

1. Find E(X), E(Y), E(XY), Cov(X,Y) and ρ( X ,Y).
2. The joint distribution of two discrete random variables X and Y is given below :

|  |  |  |  |
| --- | --- | --- | --- |
| Y  X | -3 | 2 | 4 |
| 1 | 0.1 | 0.2 | 0.2 |
| 3 | 0.3 | 0.1 | 0.1 |

Find the marginal distributions of X and Y, Cov(X,Y) and ρ( X ,Y).. Also verify that X and Y are stochastically independent.

module- V-SAMPLING THEORY &DISTRIBUTIONS, STOCHASTIC PROCESS

1. **E**xplain the following terms : (i) Null Hypothesis (ii) Confidence limits (iii) Type I and Type II Errors (iv) Level of significance (v) Standard error (vi) Test of significance
2. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. On the assumption of random throwing, do the data indicate that the die is biased ?
3. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.
4. A die is thrown 324 times and an odd number turned up 181 times. Is it reasonable to think that the die is an unbiased one ?
5. A die is tossed 960 times and it falls with 5 upwards 184 times. Can the die be considered fair at 0.01 level of significance ?
6. Find the range of number of heads out of 64 tosses of a coin which will ensure fairness of coinat 5 % level of significance using binomial distribution.
7. Find the probability that in 100 tosses of a fair coin between 45%and 55% of the outcomes are heads.
8. A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable 99% confidence limits to the proportion of foggy days in the district ?
9. A mechanist is making engine parts with axle diameter of 0.7 inches. A random sample of 10 parts showed a mean of 0.472 inches with a standard deviation of 0.04 inches. On the basis of this sample, can it be concluded that the work is inferior at 5% level of significance ?
10. The mean life time of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the life time of all the bulbs produced by the company, test the hypothesis that μ = 1600 hours against the alternate hypothesis μ≠ 1600 hours using a level of significance 0.01.
11. A random sample of 100 recorded deaths in past year showed an average life span of 71.8 years. Assuming a population with standard deviation of 8.9 years, does the data indicate that the average life span today is greater than 70 years ? Use a 0.05 level of significance.
12. In a city A, 20% of a random sample of 900 school boys has a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant ?
13. The weights of workers in a large factory are normally distributed with mean of 68kgs and standard deviation 3 kgs. If 80 samples consisting of 35 workers each are chosen, how many of the 80 samples will have the mean between 67 and 68.25 kgs ? (A(2) = 0.4772 & A(0.5) = 0.1915)
14. Find the probability that in 100 tosses of a fair coin

* Between 45% and 55% will be heads,
* 4/7 or more will be heads.

1. Out of 1000 samples of 200 children each, in how many would you expect to find that

* Less than 40% are boys
* Between 40% and 60% are boys
* 55% or more are girls

1. A random sample of 400 items chosen from an infinite population is found to have a mean of 82 and a standard deviation of 18. Find the 95% confidence limits for the mean of the population from which the sample is drawn.
2. A random sample of 100 factory workers in a large city revealed a mean weekly earnings of Rs.487 with a standard deviation of Rs. 48. With what level of confidence can we assert that the average weekly salary of all factory workers in the city is between Rs. 472 and Rs. 502 ?
3. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (Given t0.05(9) = 2.262)
4. The nine items of a sample have the following values : 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5 ? (Given t0.05(8) = 2.31)
5. A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure : 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4(in appropriate units). Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use t0.05(11) = 2.201.
6. Find the student’s “t” for the following variable values in a sample of eight : -4, -2, -2, 0, 2, 2, 3, 3 taking the mean of the universe as zero.
7. A sample of 12 measurements of the diameter of metal ball gave the mean 7.38 mm with standard deviation1.24mm. Find (i) 95% and (ii) 99% confidence limits for actual diameter given t0.05(11) = 2.201 and t0.01(11) = 3.11.
8. Two horses A and B were tested according to time to run a particular race with the following results:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Horse A | 28 | 30 | 32 | 33 | 33 | 29 | 34 |
| Horse B | 29 | 30 | 30 | 24 | 27 | 29 |  |

Test whether you can discriminate between the two horses.(t0.05 = 2.2 and t0.02= 2.72 for 11 d.f)

1. Eleven school boys were given a test in drawing. They were given a month’s further tution and a second test of equal difficulty was held at the end of it. Do the marks give evidence that the students have benefitted by extra coaching ? (Given t0.05(10) = 2.228)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Boys | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Marks(I Test) | 23 | 20 | 19 | 21 | 18 | 20 | 18 | 17 | 23 | 16 | 19 |
| Marks(II Test) | 24 | 19 | 22 | 18 | 20 | 22 | 20 | 20 | 23 | 20 | 17 |

1. A random sample of 10 boys had the following I.Q. : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100 at 5% level of significance ?
2. A set of five similar coins is tossed 320 times and the result is :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| No. of heads | 0 | 1 | 2 | 3 | 4 | 5 |
| Frequency | 6 | 27 | 72 | 112 | 71 | 32 |

Test the hypothesis that the data follows a binomial distribution.

1. Fit a Poisson distribution to the following data and test for its goodness of fit at 5 % level of significance.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 |
| f | 419 | 352 | 154 | 56 | 19 |

(Given χ20.05  forγ = 3 is 7.82).

1. A survey conducted on 64 families with 3 children each are recorded as follows :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No. of male children | 0 | 1 | 2 | 3 |
| No. of families | 6 | 19 | 29 | 10 |

Apply Chi- square test to test whether male and female children are equiprobable at 5% level of significance. (Given χ20.05  for γ = 2 is 7.82).

1. For the following data, test the hypothesis that the accidents are uniformly distributed over all the days of the week for 99% confidence

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Day | Sun | Mon | Tue | Wed | Thu | Fri | Sat | Total |
| No. of accidents | 14 | 16 | 8 | 12 | 11 | 9 | 14 | 84 |

1. Define stochastic matrix. Find the unique fixed probability vector for the matrix :

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1. Show that P = is a regular stochastic matrix. Also find the associated unique

fixed probability vector.

1. Prove that the Markov chain whose transition probability matrix given by

is irreducible. Also find the corresponding stationary probability vector.

1. A student’s study habits are as follows : If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study ?
2. Each year a man trades his car for a new car in three brands of the popular company MarutiUdyog Ltd. If he has a ‘Standard’ he trades it for ‘Zen’. If he has a ‘Zen’ he trades it for an ‘Esteem’. If he has an ‘Esteem’ he is just as likely to trade it for a new Esteem or for a Zen or a Standard one. In 1996, he bought his first car which was Esteem. Find the probability that he has (i) 1998 Esteem (ii) 1998 Standard (iii) 1999 Zen (iv) 1999 Esteem. In the long run how often will he have an Esteem.
3. Three boys A, B and C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball.
4. A software engineer goes to his office everyday by motorbike or by car. He never goes by bike on two consecutive days, but if he goes by car on a day then he is equally likely to go by car or by bike the next day. Find the t.p.m. of the Markov chain. If car is used on the first day of the week find the probability that after four days (i) bike is used (ii) car is used.
5. Explain (i) Transient state (ii) Recurrent state (iii) Absorbing state of a Markov chain.

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